International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 6, Issue 1, Dec – Jan 2017; 43-54

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

ANALYSIS OF TWO ECHELON INVENTORY SYSTEM WITH JOINT ORDERING POLICY

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ABSTRACT

Inventory is essential for the efficient running of any business. Single location inventory system is considered by many researchers. This paper deals with two echelon inventory system with handling two products having joint ordering policy. The demand for the products follows independent poison distributions at retailer and distributor node. The items are supplied to the retailers from the distribution center (DC) administrated with exponential lead time having parameter μ (>0). The joint probability disruption of the inventory levels of two products at retailer and the supplier are obtained in the steady state case. Various system performance measures are derived and the long run total expected inventory cost rate is calculated. Several instances of numerical examples, which provide insight into the behavior of the system, are presented.

KEYWORDS: Continuous Review Inventory System, Two-Echelon, Joint Ordering Policy

1. INTRODUCTION

Inventory control system in two-echelon or multi echelon has been considered by many researchers. Most of the papers the authors concentrate on periodic review inventory system. A first quantitative model to find an optimal (s, S) inventory policies for single item at single location is considered by Veinott and Wagner[10]. Thangaraj and Ramanarayanan[9] develop an inventory model with two re-order levels and random lifetime. Gross and Harris [7] considered the inventory system with state dependent lead times. All the above papers deals only with single location inventory system. This paper deals with two echelon inventory system(retailer –lower echelon, distributor – higher echelon) handling two different products supplied by the same distributor.

Anbazhagan and Arivarignan [1] have analyzed two commodity inventory system under various ordering policies. Yadavalli et. al., [11] have analyzed a model with joint ordering policy and varying order quantities. Yadavalli et. al., [12] have considered a two commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

In a very recent paper, Anbazhagan et. al. [2] considered analysis of two commodity inventory system with compliment for bulk demand in which, one of the items for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item is also assumed when the items are out of stock. The above model is studied only at single location(Lower echelon). We extend the same in to multi-echelon structure (Supply Chain)with joint ordering policy. The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done:

Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. Section 6 provides Numerical examples and sensitivity analysis.

2. MODEL

2.1. The Problem Description

The inventory control system considered in this paper is defined as follows. We assume that finished products are supplied from warehouse to distribution centre (DC) which adopts (0, M) replenishment policy then the product is supplied to retailer (R) who adopts (s_i, Q_i) policy. The demands at retailer node follows independent Poisson distribution with rate λ_i (i = 1, 2). The items are supplied to the retailers in packs of Q ($Q = Q_1+Q_2$) where Q_i ($= S_i-s_i$) from the distribution center (DC) administrated with exponential lead time having parameter μ (>0). The direct demand at distributor node follows Poisson distribution with rate λ_D. The replenishment of items in terms of pockets is made from WH to DC is instantaneous. Demands that occur during the stock out periods are assumed to be lost sales. In this model the maximum inventory level at retailer node S_i is fixed and he reorder level is fixed as s_i for the i-th commodity and the ordering policy is to place order for Q_i ($= S_i-s_i$) items (i = 1, 2) when both the inventory levels are less than or equal to their respective reorder levels. The maximum inventory level at DC is M (M = nQ). The joint probability distribution for both commodities is obtained in steady state cases.

The optimization criterion is to minimize the total cost rate incurred at all the location subject to the performance level constrains. According to the assumptions the on hand inventory levels at all the nodes follows a random process.

2.2. Analysis

Let $I_i(t)$, (i = 1, 2,3) denote the on-hand inventory levels for commodity-1, commodity-2 at retailer node and Distribution Centre (DC) respectively at time t+. From the assumptions on the input and output processes, $I(t) = \{ I_i(t);\ t\geq 0\}$(i = 1, 2,3) is a Markov process with state space

$$E = \left\{ \begin{array}{l} (i,k,m)\ \ / \ i = S_1,(S_1-1),\ \ldots\ ,s_1,(s_1-1),\ \ldots\ ,2,1,0.\ , \\ k\ = S_2,(S_2-1),\ \ldots\ ,s_2,(s_2-1),\ \ldots\ ,2,1,0. \\ m\ = \ nQ,(n\text{-}1)Q,\ \ldots\ ,Q \end{array} \right\}$$

Since E is finite and all its states are recurrent non-null, $I(t) = \{ I_i(t);\ t \geq 0\}$is an irreducible Markov process with state space E and it is an ergodic process. Hence the limiting distribution exists and is independent of the initial state. The infinitesimal generator of this process

$$R = (a(i,k,m:j,l,n))_{(i,k,m),(j,l,n)\in E}$$

can be obtained from the following arguments.

- The arrival of a demand for commodity-1 at retailer node makes a state transition in the Markov process from (i, k, m) to (i-1, k, m) with intensity of transition λ_1.
- The arrival of a demand for commodity-2 at retailer node makes a state transition in the Markov process from (i, k, m) to (i, k-1 m) with intensity of transition λ_2.

- The arrival of a demand at distributor node makes a state transition in the Markov process from (i, k, m) to (i, k m-Q) with intensity of transition λ_D.
- Joint Replenishment of inventory for commodity-1and commodity-2 at retailer node makes a state transition in the Markov process from (i, k, m) to $(i+Q_1, k+Q_2, m-Q)$ with intensity of transition μ .where $Q = Q_1+Q_2$

The infinitesimal generator R is given by

$$R = \begin{pmatrix} A & B & 0 & \cdots & 0 & 0 \\ 0 & A & B & \cdots & 0 & 0 \\ 0 & 0 & A & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & B \\ B & 0 & 0 & \cdots & 0 & A \end{pmatrix}$$

The entities of matrix R are given by

$$[R]_{pxq} = \begin{cases} A \quad \text{if} \quad p = q & ;\ p = nQ\ ,(n\text{-}1)Q,\ \ldots\ ,3Q,\ 2Q. \\ B \quad \text{if} \quad p = q+Q & ;\ p = nQ\ ,(n\text{-}1)Q,\ \ldots\ ,3Q,\ 2Q. \\ B \quad \text{if} \quad p = q-(n-1)Q & ;\ p = Q. \\ 0 \quad \text{otherwise} & \end{cases}$$

The sub matrices of matrix R are given by

$$[A]_{pxq} = \begin{cases} A_1 \quad \text{if} \quad p = q & ;\ p = S_2, (S_2-1), (S_2-2),\ \ldots\ ,(s_2+1). \\ A_2 \quad \text{if} \quad p = q+Q & ;\ p = S_2, (S_2-1), (S_2-2),\ \ldots\ ,1. \\ A3 \quad \text{if} \quad p = q & ;\ p = s_2,\ (s_2-1), ((s_2-2),\ \ldots\ 1. \\ A4 \quad \text{if} \quad p = q & ;\ p = 0 \\ 0 \quad \text{otherwise} & \end{cases}$$

$$[B] = \begin{cases} \mu \quad \text{if } p = q + Q_2\ ;\ q = s_2\ ,(s_2-1),\ \ldots\ ,1,0 \\ 0 \quad \text{otherwise} \end{cases}$$

The sub matrices of matrix A are given by

$$A_1 = \begin{cases} \lambda_1 & \text{if } p = q\text{-}1\ ;\ q = 1,2,\ \ldots\ ,S_1 \\ -(\lambda_1+\lambda_2+\lambda_D) & \text{if } p = q\text{-}1\ ;\ q = 1,2,\ \ldots\ ,S_1 \\ -(\lambda_2+\lambda_D) & \text{if } p = q\ ;\ q = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$A_2 = \begin{cases} \lambda_2 & \text{if } p = q\text{-}1 \text{ ; } q = 1,2, \dots , S_1 \\ 0 & \text{otherwise} \end{cases}$$

$$A_3 = \begin{cases} \lambda_1 & \text{if } p = q\text{-}1 \text{ ; } q = 1,2, \dots , S_1 \\ -(\lambda_1+\lambda_2+\lambda_D+\mu) & \text{if } p = q\text{-}1 \text{ ; } q = 1,2, \dots , S_1 \\ -(\lambda_2+\lambda_D+\mu) & \text{if } p = q \quad ; q = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$A_4 = \begin{cases} \lambda_1 & \text{if } p = q\text{-}1 \text{ ; } q = 1,2, \dots , S_1 \\ -(\lambda_1+\lambda_2+\lambda_D+\mu) & \text{if } p = q\text{-}1 \text{ ; } q = 1,2, \dots , S_1 \\ -(\mu+\lambda_D) & \text{if } p = q \quad ; q = 0 \\ 0 & \text{otherwise} \end{cases}$$

4.2.3 Transient Analysis

Define the transient probability function

$p_{(i,\,k,\,m)}(j,\,l,\,n : t) = p_r\,\{(I_1(t), I_2(t), I_3(t)) = (j, l, n) \mid (I_1(0), I_2(0), I_3(0)) = (i, k, m)\}$.

The transient matrix for $t \geq 0$ is of the form $P(t) = (p_{(i,\,k,\,m)}(j,\,l,\,n : t))_{(i,\,k,\,m)(j,\,l,\,n)\in E}$ satisfies the Kolmogorov-forward equation $P'(t) = P(t).R$, where R is the infinitesimal generator of the process $\{I(t), t \geq 0\}$. From the above equation, together with initial condition $P(0) = I$, the solution can be express of the form $P(t) = P(0)e^{Rt} = e^{Rt}$, where the matrix expansion in power series form is

$$e^{Rt} = I + \sum_{n=1}^{\infty} \frac{R^n t^n}{n!}.$$

4.2.4 Steady State Analysis

The structure of the infinitesimal matrix R reveals that the state space E of the Markov process $\{I(t); t \geq 0\}$ is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$$^{m}\Pi_i^k = \lim_{t\to\infty} \Pr\left\{ \left(I_1(t), I_2(t), I_D(t)\right) = \left(i,k,m\right) \right\},$$

where $^{m}\Pi_i^k$ is the steady state probability that the system be in state (i,k,m), (Cinlar [5]).

Let $\Pi = ({}^{nQ}\Pi, {}^{(n-1)Q}\Pi, {}^{(n-2)Q}\Pi, {}^{2Q}\Pi, {}^{Q}\Pi.\ )$ denote the steady state probability distribution, where ${}^{jQ}\Pi = \left(\Pi_S^k, \Pi_{S-1}^k, \dots, \Pi_1^k, \Pi_0^k\right)$ for j = 1, 2 ... n and k = 1,2,…, S for the system under consideration. For each (i,k,m), $^{m}\Pi_i^k$ can be obtained by solving the matrix equation $\Pi R = 0$

Since the state space is finite and R is irreducible, the stationary probability vector Π for the generator R always exists and satisfies

ΠR=0 and **Πe=1**

The vector Π can be represented by

$$\Pi = (\Pi^{<Q1,Q2>}, \Pi^{<2Q1,2Q2>}, \Pi^{<3Q1,3Q2>}, ..., \Pi^{<n1Q1,n2Q2>}$$

Now the structure of R shows, the model under study is a finite birth death model in the Markovian environment. Hence we use the Gaver algorithm for computing the limiting probability vector. For the sake of completeness we provide the algorithm here.

4.2.5 Algorithm

1. Determine recursively the matrix Dn, $0 \leq n \leq N$ by using

$$D0 = A_0$$

$$Dn = A_n + B_n(-D_{n-1}^{-1})\,C, \quad n = 1, 2, ...K$$

2. Slove the system

$$\Pi^{<N>} D_N = 0$$

3. Compute recursively the vector $\Pi^{<n>}, \quad n = N-1, ..., 0$ using

$$\Pi{<}n{>} = \Pi^{<n+1>} B_{n+1}(-D_n^{-1}), n = n-1, ..., 0$$

4. Re-normalize the vector Π, using

$$\Pi e = 1.$$

4.2.6. Operating Characteristics

In this section, we derive some important system performance measures.

(a) Mean Inventory Level

Let IL_{R1} and IL_{R2} denote the expected inventory level in the steady state at retailer node for the commodity-1 and commodity-2, and IL_D denote the expected inventory level at distribution centre. They are defined as

$$IL_{R1} = \sum_{m=Q}^{nQ}{}^{*}\sum_{k=0}^{S_2}\sum_{i=0}^{S_1} i\Pi^{<<<i,k,m,o>>>} \tag{4.1}$$

$$IL_{R2} = \sum_{m=Q}^{nQ}{}^{*}\sum_{k=0}^{S_2}\sum_{i=0}^{S_1} k\Pi^{<<<i,k,m,o>>>} \tag{4.2}$$

$$IL_D = \sum_{m=Q}^{nQ} {}^{*} \sum_{k=0}^{S_2} \sum_{i=0}^{S_1} m\Pi^{<<<i,k,m,o>>>} \; ; Q = Q_1+Q_2 \tag{4.3}$$

(b) Mean Reorder Rate

Consider the reorder events β_R for both commodities at retailer node and β_D of at distribution centre. It is observe that β_D event occur whenever the inventory level at DC node reaches 0 whereas the event β_R occurs whenever the inventory level at retailer node drops to either (s_1,s_2) or $(s_1,j),j < s_2$ or $(i,s_2),i < s_1$.

The mean reorder rate at retailer node and distribution centre are given by

$$B_R = \sum_{m=Q}^{nQ} \left(\sum_{k=0}^{s_2} (\lambda_1)\Pi^{<<<<s_1+1,k,m>>>>} + \sum_{i=0}^{s_1} (\lambda_2)\Pi^{<<<i,s_2+1,m>>>} \right) \tag{4.4}$$

$$B_D = \sum_{k=0}^{s_2} \sum_{i=0}^{s_1} (\mu + \lambda_D)\Pi^{<<<<i,k,Q>>>>} \tag{4.5}$$

(c) Mean Shortage Rate

Shortage occurs only at retailer node and the mean shortage rate at retailer is denoted by α_R which is given by

$$\alpha_R = \sum_{m=Q}^{nQ} {}^{*} \left(\sum_{k=0}^{S_2} (\lambda_1)\Pi^{<<<0,k,m>>>} + \sum_{k=0}^{S_1} (\lambda_2)\Pi^{<<<i,0,m>>>} \right) \tag{4.6}$$

4.2.7 Cost Analysis

In this section we analyze the cost structure for the proposed models by considering the minimization of the steady state total expected cost per time.

The long run expected cost rate for the model is defined to be

$$C(s,Q) = h_{R1}\, IL_{R1} + h_{R2}\, IL_{R2} + h_D\, IL_D + k_r\, B_R + k_D\, B_D + g_R\, \alpha_R \tag{4.7}$$

where

- $\mathbf{h_{R1}}$, $\mathbf{h_{R2}}$, and $\mathbf{h_D}$ are the holding cost per unit of item of Commodity- 1, Commodity-2 at retailer node and holding cost per unit items (Q items, $Q = Q_1+Q_2$) at distribution centre respectively per unit time.
- $\mathbf{IL_{R1}}$, $\mathbf{IL_{R2}}$, and $\mathbf{IL_D}$ are the inventory level of Commodity- 1, Commodity- 2 at retailer node and inventory level at distribution centre respectively per unit time.
- $\mathbf{K_r}$**, and** $\mathbf{K_D}$ are the Fixed ordering cost at retailer node distribution centre respectively.
- $\mathbf{B_R}$ and $\mathbf{B_D}$ are the mean reorder rate at retailer node and distribution centre.
- $\boldsymbol{\alpha_R}$ is the mean shortage rate at retail node.

- **g_R** is the shortage cost per unit shortage at retailer node.

Although, we have not proved analytically the convexity of the cost function ***TC(s₁,s₂ Q)*** ,our experience with considerable number of numerical examples indicates that ***TC(s₁,s₂ Q)*** for fixed Q to be convex in s. In some cases, it turned out to be an increasing function of s. Hence, we adopted the numerical search procedure to determine the optimal values s*, consequently, we obtain optimal n*. For large number of parameters, our calculation of ***TC(s₁,s₂ Q)*** revealed a convex structure for the same.

5. NUMERICAL ILLUSTRATION

In the section the problem of minimizing the long run total expected cost per unit time under the following cost structure is considered for discussion. The optimum values of the system parameters s_1 and s_2 are obtained and the sensitive analysis is also done for the system.

The results we obtained in the steady state case may be illustrated through the following numerical example,

$S_1=20$, $S_2 = 25$, $M= 200$, $\lambda_1 = 3, \lambda_2 = 4$, $\lambda_D = 2$, $\mu = 3$ $h_R = 1.1$, $h_D = 1.2$, $k_R = 1.5, k_d = 1.3$ $g_R = 2.1$, $g_D = 2.2$, $C_o = 2.3$.

The cost for different reorder level are given by

Table 1: Total Expected Cost Rate as a Function S_1, S_2 and Q

s_1	2	3	4	5*	6	7	8
s_2	3	4	5	6*	7	8	9
$Q=(Q_{1,+}Q_2)$	40	38	36	34	32	30	28
$TC(s_1,s_2\ Q)$	115.7454	105.8264	90.5506	87.90082*	94.28986	110.392	123.6665

For the inventory capacity S, the optimal reorder level s* and optimal cost ***TC (s₁, s2, Q)*** are indicated by the symbol *. The Convexity of the cost function is given in the graph.

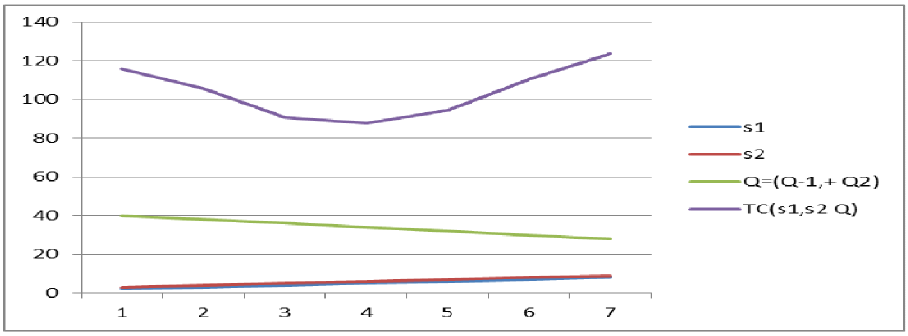

Figure 1: Total Expected Cost Rate as a Function *TC (S₁, S2, Q)*, S_1, S_2 and Q

1.1. Sensitivity Analysis

The effect of changes in Demand rate at retailer node for product 1 and 2.

Table 2: Total Expected Cost Rate as a Function When Demand Increases

	$\lambda_2 = 8$	$\lambda_2 = 10$	$\lambda_2 = 12$	$\lambda_2 = 14$	$\lambda_2 = 16$
$\lambda_1 = 12$	45.073539	46.419603	47.214733	47.629158	47.785721
$\lambda_1 = 14$	46.271834	47.617898	48.413028	48.827452	48.984016

λ_1 = 16	47.136498	48.482562	49.277692	49.692117	49.848680
λ_1 = 18	47.793130	49.139194	49.934324	50.348749	50.505312
λ_1 = 20	48.325884	49.671948	50.467078	50.881502	51.038066

The graph of the demand rate variation is given below and it describes, if the demand rate increases then the total cost also increases.

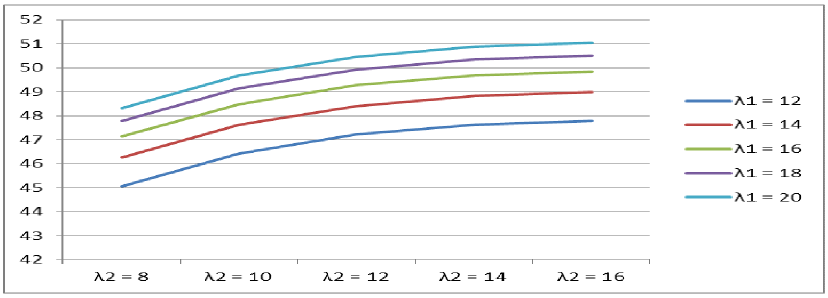

Figure 2: *TC (S_1, S2, Q)* for Different Demand Rates

Table 3: Total Expected Cost Rate as a Function When S_1 and S_1 Increases

	S_1=2	**S_1=4**	**S_1=6**	**S_1=8**	**S_1=10**
S_1=45	111.998164	118.761642	124.799813	130.664478	136.843108
S_1=50	118.699318	124.359054	129.299781	133.996082	138.773110
S_1=55	123.090318	127.562168	131.434202	135.090008	138.765317
S_1=60	125.576395	129.128145	132.243072	135.192545	138.137225
S_1=65	126.984419	130.065937	132.851239	135.511516	138.145444

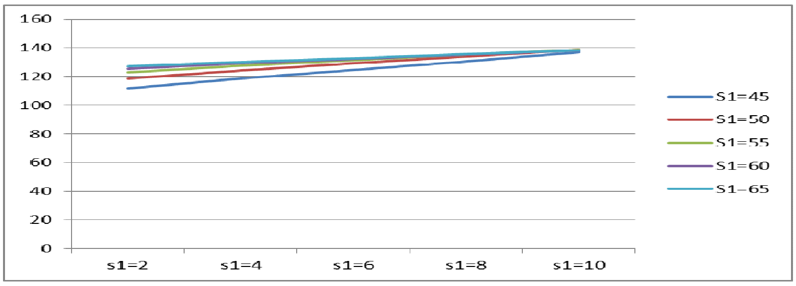

Figure 3: *TC (S_1, S2, Q)* For Different S_1 and S_1 Values

From the graph it is identified that the total cost increases when the s and S increases.

Table 4: Total Expected Cost Rate as a Function When S_1 and S_2 Increases

	S_2=30	**S_2=35**	**S_2=40**	**S_2=45**	**S_2=50**
S_1=20	189.8666	179.2934	165.6356	152.2989	141.219
S_1=25	195.0113	186.1366	173.264	159.6361	147.8542
S_1=30	199.5703	192.4455	180.7208	167.1007	154.6623
S_1=35	203.6145	198.1655	187.8799	174.6251	161.6628
S_1=40	207.2345	203.2862	194.623	182.1144	168.8466

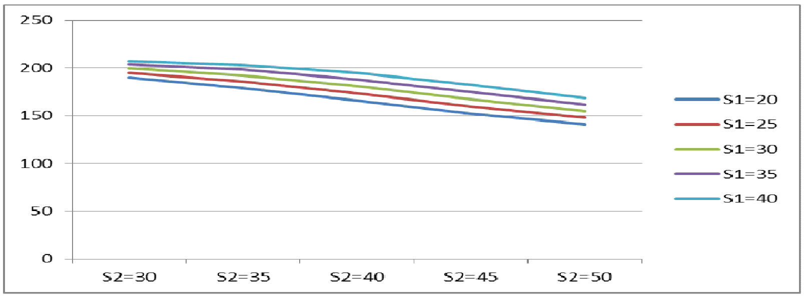

Figure 4: *TC (S_1, $S2_,$ Q)* for Different S_1 and S_2 Values

From the graph it is identified that the total cost decrease when S_2 increases and increases when S_1 increases

Table 5: Total Expected Cost Rate When H_r and H_d Increases

	h_D=0.04	h_D=0.08	h_D=0.12	h_D=0.16	h_D=0.20
h_R=0.002	43.1924942	43.87908	44.5655	45.25206	45.93862
h_R=0.004	43.2037231	43.89028	44.5767	45.26326	45.94982
h_R=0.006	43.2149519	43.90148	44.58804	45.27446	45.96102
h_R=0.008	43.2261794	43.91268	44.59924	45.28566	45.97222
h_R=0.010	43.2374082	43.92388	44.61044	45.297	45.98342

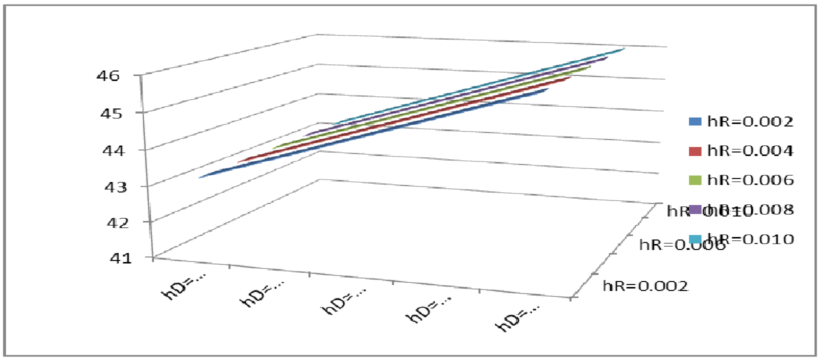

Figure 5: *TC (S_1, $S2_,$ Q)* for Different H_r and H_d Values

As is to be expected the graph shows that the total cost increases when h_R and h_D increases

Table 6: Total Expected Cost Rate When G_r and K_d Increases

	g_R=0.2	g_R=0.4	g_R=0.6	g_R=0.8	g_R=1.0
k_D=10	32.346268	33.06786	33.78945	34.51104	35.23264
k_D=15	34.204663	34.926255	35.64785	36.36944	37.09103
k_D=20	36.063058	36.78465	37.50624	38.22783	38.94943
k_D=25	37.921453	38.643045	39.36464	40.08623	40.80782
k_D=30	39.779848	40.50144	41.22303	41.94462	42.66622

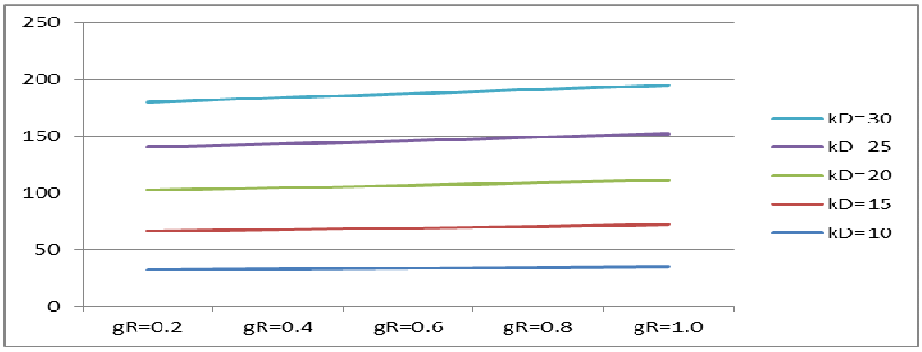

Figure 6: ***TC (S₁, S2, Q)*** **for Different G_R and K_D Values**

As is to be expected the graph shows that the total cost increases when g_R and k_D increases.

6. CONCLUSIONS

This paper deals with a two echelon Inventory system with two The demand at retailer node follows independent Poisson with rate λ_1 and λ_2. The Joint replenishment ordering policy is applied at retailer node. The direct demand at distributor is assumed to be Poisson with rate $\lambda_{D.}$ The structure of the chain allows vertical movement of goods from to supplier to Retailer. If there is no stock for product at retailer the demand is refused and it is treated as lost sale. The model is analyzed within the framework of Markov processes. Joint probability distribution of inventory levels at DC and Retailer for both products are computed in the steady state. Various system performance measures are derived and the long-run expected cost is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values on the total expected cost rate. It would be interesting to analyze the problem discussed in this paper by relaxing the assumption of exponentially distributed lead-times to a class of arbitrarily distributed lead-times using techniques from renewal theory and semi-regenerative processes. Once this is done, the general model can be used to generate various special eases.

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